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# Factors Affecting Residual Strength Prediction of a Cracked Aircraft Structure

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**Various factors influencing the analytical residual strength prediction of a cracked, stiffened structure are discussed. The influence of flexibility of attachment on crack openings, stresses in the substructure and fasteners, and J-integral values are presented. The flexibility of a fastener is shown to have considerable influence on the elastic stress intensity factors. The J-integral values computed by assuming an elastic, Dugdale-type strip plastic zone, and Prandtl-Reuss material behavior are compared. The analytical results of crack openings and strains in the stiffeners are compared with experimental results.**

## Introduction

DAMAGE tolerance requirements, as outlined in MIL-A-83444, specify that a residual strength analysis be performed on all structures critical to flight safety. If a given arrangement is known to have a propensity to fracture in a plane strain mode, the tools of linear elastic fracture mechanics can be used with a high degree of confidence and success. However, once material thickness, toughness, or structural arrangements are such that prior to fracture extensive amounts of plasticity and accompanying slow tear occur, modifications to, or extensions of, existing linear analyses must be formulated. This new method must be such that designers and stress analysts can take advantage of tougher materials and be able to predict not only critical stress but crack size at fracture as well. One such method, using the material resistance curve ($\sqrt{J}$ vs crack extension), in conjunction with elastic-plastic analysis of the cracked structure, was recently developed by Ratwani and Wilhem.[1-3] The application of this technique to residual strength prediction of stiffened panels is demonstrated in Refs. 1 and 2. The application of this technique to cracked pressure vessels was reported by the authors in Ref. 3, where good correlation was obtained between analytically predicted residual strength values and experimental results.

The residual strength prediction is affected by several factors, such as metallurgical, assumptions in the analysis, loading conditions, and crack tip buckling. A detailed study was carried out in Refs. 1 and 2 to investigate the influence of various factors on analytical residual strength predictions. The residual strength prediction based on two possible failure cases, namely, skin-critical and stiffener-critical, were studied in those references. Some of the important factors influencing the residual strength prediction are discussed in this paper.

## Influence of Fastener Modeling

In the analysis of any structure, the assumptions made should be consistent with the actual behavior of the loaded structure. The proper boundary conditions and load transfer effects should be taken into consideration. These factors are particularly important in cracked structures because of the presence of singular stresses ahead of a crack tip. In the analysis of a cracked, stiffened structure, an important consideration is the flexibility of the fastener connecting the stiffener to the skin.[1,2,4] Consider the wing channel panel geometry shown in Fig. 1. The method of modeling this structure is illustrated in Fig. 2. A portion of the stiffened skin shown in Fig. 2a is viewed as consisting of skin and stiffener connected by rivets, as shown in Fig. 2b. The skin or sheet is idealized as a plate element and the connecting fasteners as shear elements (Fig. 2c). The connected portions of the stiffener, i.e., hat portion and flange, are idealized as bar or rod elements and the stiffener webs are idealized as membrane elements. The rod elements are considered to have the same cross-sectional area as that of the two flanges. The membrane elements in the model have the same thickness as the two webs. The shear elements representing fasteners may be modeled in two ways. In the first model, the area of the shear elements is kept the same as the area of the fastener. This model is termed as the equivalent model.[1] In the second model, termed as the flexible fastener model, it is assumed that the deflection of the shear element is the same as the experimentally determined deflection of a similarly attached joint. In Ref. 5, Swift has presented the following empirical equation for the rivet shear deflection in riveted aluminum

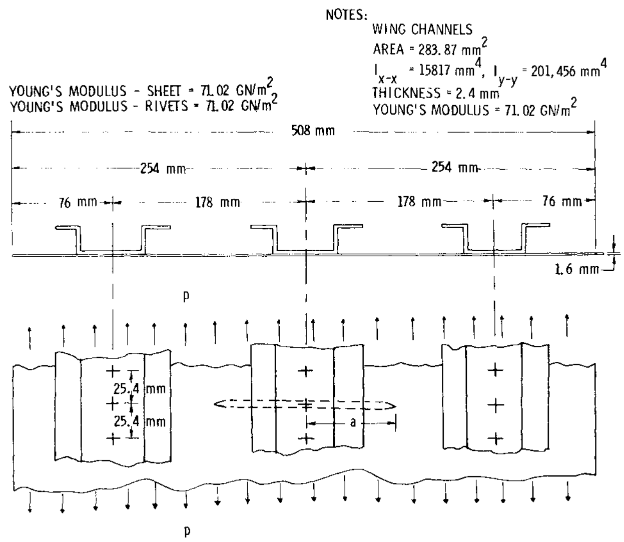


Fig. 1 Wing channel stiffened panel geometry.

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alloy sheets:

$$\delta = Pf/E_{al}d \tag{1}$$

where $\delta$ = deflection, $P$ = applied load, $E_{al}$ = modulus of aluminum, and $d$ = rivet diameter and

$$f = 5.0 + 0.8\left(\frac{d}{t_1} + \frac{d}{t_2}\frac{E_1}{E_2}\right) \tag{2}$$

where $t_1$ and $t_2$ are the thicknesses of the two joined sheets and $E_1$ and $E_2$ are the respective Young's moduli.

In the idealized shear element, the deflection is given by

$$\delta = Ph/A_sG_{al} \tag{3}$$

where $A_s$ = the area of the shear element, $G_{al}$ = shear modulus of aluminum, $h$ = distance between the centers of the sheets that are connected.

Equating the two deflections [Eqs. (1) and (3)],

$$Pf/E_{al}d = Ph/A_sG_{al} \tag{4}$$

For the wing panel geometry of Fig. 1, the rivet diameter is 6.35 mm (one-quarter inch) and rivet pitch, 25.4 mm (1 in.). For this geometry,

$$E_1 = E_2 = 71.02 \text{ GN/m}^2$$

$$t_1 = 1.6 \text{ mm} \quad \text{and} \quad t_2 = 2.4 \text{ mm}$$

Substituting in Eq. (2) gives $f = 10.308$.

The distance between the centers of the connected sheets is:

$$h = (1.6 \text{ mm} + 2.4 \text{ mm})/2 = 2 \text{ mm}$$

The area of the shear element given by Eq. (4) is ($\nu$ = Poisson's ratio)

$$A_s = 2(1+\nu)\frac{hd}{f} = 2(1+0.33)\times\frac{(2)(6.35)}{10.308} = 3.27 \text{ mm}^2$$

If $h_l$ is the distance between node points in the finite-element model where a shear element is provided, then the thickness of the shear element in the model is given by $t = A_s/h_l$. Generally, six to eight rivets above the crack plane are modeled as individual rivets, represented by shear elements, as these are primarily effective in load transfer between sheet and stiffener. Beyond these rivets the stiffener may be assumed to be continuously attached to the skin, and in computing the stiffness of shear elements beyond (6-8 rivets), the distance $h_l$ is taken as the distance between consecutive grid points along the line of attachment. In the actual finite-element analysis, the NASTRAN computer program was used.

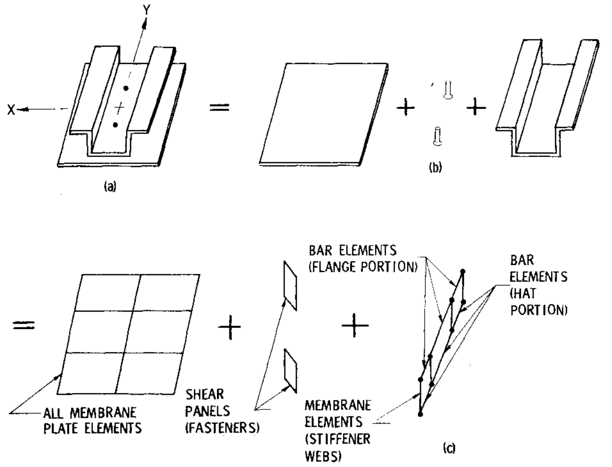


**Fig. 2 Sheet-rivet-channel finite-element idealization.**

**Table 1 Influence of fastener model on stress intensity factors**

| Half-crack length a, mm (in.) | Stress intensity factor $K_I/(\sigma\sqrt{\pi a})$ Equivalent area model | Flexible fastener model | Percentage difference |
|---|---|---|---|
| 71.1 (2.8) | 0.587 | 0.667 | 13.6 |
| 137.0 (5.4) | 0.620 | 0.678 | 9.4 |

### Effect of Fastener Modeling on Crack Opening and Stress Intensity Factors

Figure 3 shows the effect of fastener modeling on crack surface displacements for the two fastener models at half-crack lengths of 71.1 and 137 mm (2.8 and 5.4 in.). It is seen that for the flexible fastener model, crack surface displacements are more than 50% higher than the corresponding displacements for the equivalent area model. These crack surface displacements have considerable effect on the resulting stress intensity factors as shown in Table 1.

### Influence of Fastener Modeling on Stiffener Stresses

The effect of fastener modeling on the stress in the outer and central stiffener is shown in Fig. 4 for the 71.1- and 137-mm (2.8- and 5.4-in.) crack lengths. It is seen that the stresses in the outer stiffener are only slightly influenced by the type of fastener model used, since the crack has not yet reached the outer stiffener and there are no crack surface displacements at the outer stiffener. However, when the crack tip is beyond the outer stiffener, the crack surface displacements at the location of the outer stiffener and stresses in the outer stiffener are definitely affected by the fastener modeling. In Fig. 4, it is seen that the stress in the central stiffener is significantly affected by the fastener modeling. Values for the flexible fastener model are about 9% lower than those obtained for the equivalent area model.

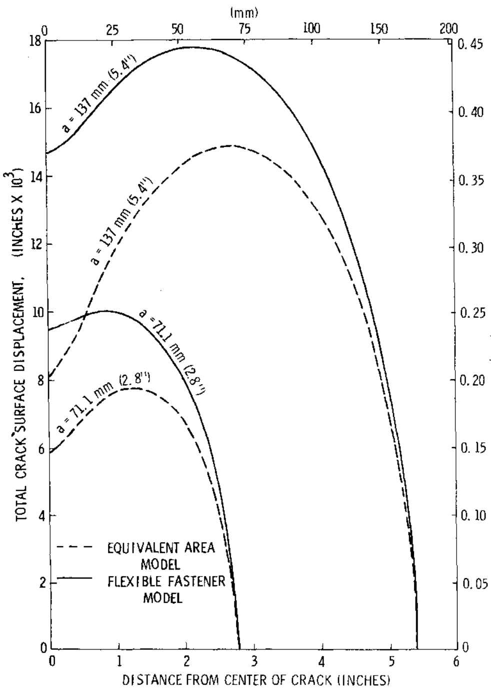


**Fig. 3 Crack surface displacements for two fastener models.**

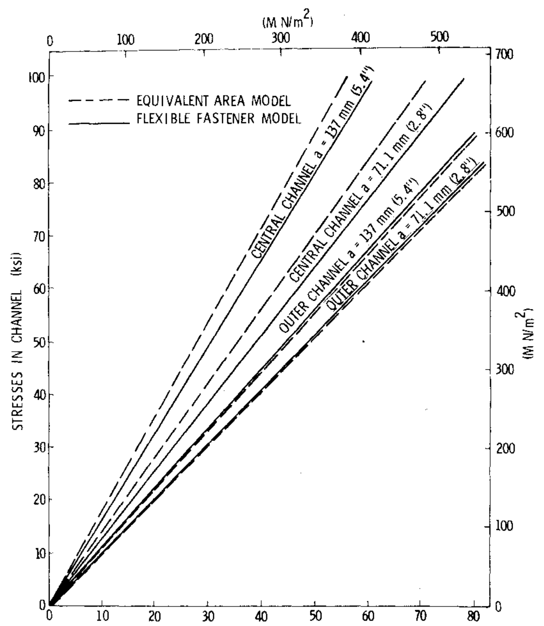


**Fig. 4 Stresses in wing channel as a function of applied stress.**

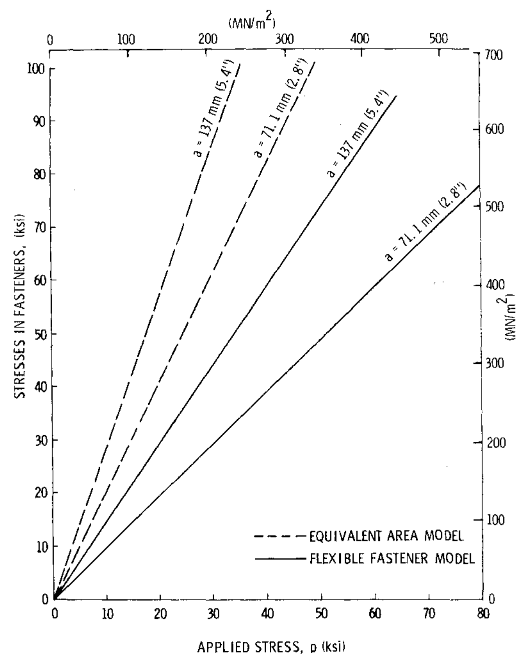


**Fig. 5 Shear stresses in central channel fastener above crack plane for two fastener models.**

### Influence of Fastener Modeling on Fastener Stresses

The influence of two fastener models on fastener shear stresses in the central channel, assuming elastic fastener behavior for the same two crack lengths, is shown in Fig. 5. It is seen that the stresses given by the equivalent area model are extremely high and unrealistic. For example, at an applied stress of 206.85 $MN/m^2$ (30 ksi), the equivalent area model gives stresses almost twice as high as those given by the flexible fastener model. For a 71.1-mm (2.8-in.) half-crack length and an applied stress of 275.80 $MN/m^2$ (40 ksi), the calculated fastener stress is close to the ultimate stress. The testing of this particular wing channel panel showed it was able to sustain much higher loads without fastener failure. Hence, the fastener stresses given by the equivalent area model are unrealistic.

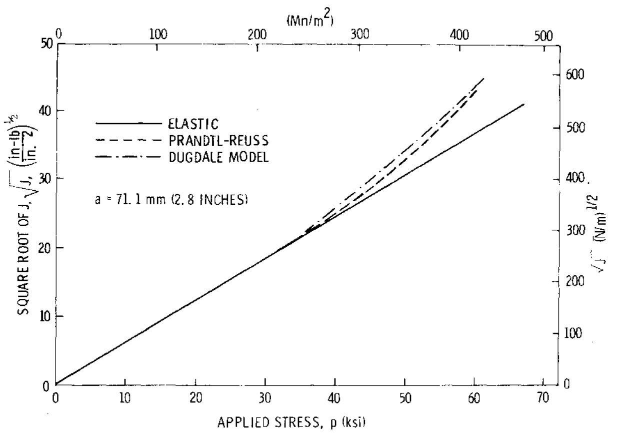


**Fig. 6 Comparison of $\sqrt{J}$ for elastic, Prandtl-Reuss, and Dugdale model material behavior.**

## Influence of Elastic and Elastic-Plastic Analyses

The analysis of a structure is generally based on either linear elastic or elastic-plastic material behavior. For cracked structures, the commonly used elastic-plastic analyses are based on a Dugdale-type strip plastic zone,[2] or assuming a Prandtl-Reuss material behavior. The panel geometry shown in Fig. 1 was analyzed using elastic and elastic-plastic analyses, and the results are discussed in this paper.

### Influence of Analysis Method on $\sqrt{J}$ Values

The $J$ integral is defined by Rice[6] as

$$J=\int_\Gamma \left(W\mathrm{d}y-\vec{T}\frac{\partial \vec{u}}{\partial x}\,\mathrm{d}s\right) \tag{5}$$

where $\Gamma$ is any contour surrounding the crack tip, traversing in a counter-clockwise direction, $W$ is the strain energy density, $\vec{T}$ is the traction on $\Gamma$, and $\vec{u}$ is the displacement on an element along arch $s$.

The strain-energy density $W$ is given by

$$W=\int [\sigma_x \mathrm{d}\epsilon_x+\tau_{xy}\mathrm{d}r_{xy}+\tau_{xz}\mathrm{d}r_{xz}+\sigma_y\mathrm{d}\epsilon_y+\tau_{yz}\mathrm{d}r_{yz}+\sigma_z\mathrm{d}\epsilon_z] \tag{6}$$

and for generalized plane stress

$$W=\int [\sigma_x\mathrm{d}\epsilon_x+\tau_{xy}\mathrm{d}r_{xy}+\sigma_y\mathrm{d}\epsilon_y] \tag{7}$$

For elastic material behavior, $J$ is equivalent to Irwin's strain-energy release rate $\mathcal{G}$. For Mode I, the relation between $\mathcal{G}$ and stress intensity factor $K_I$ is given by

$$\mathcal{G}=\frac{1-\nu^2}{E}K_I{}^2 \quad \text{for plane strain}$$

$$\mathcal{G}=\frac{K_I{}^2}{E} \quad \text{for plane stress} \tag{8}$$

The variation of the square root of $J(\sqrt{J})$ with applied stress, $p$, for a half-crack length of 7.1 mm (2.8 in.) is shown in Fig. 6. The $\sqrt{J}$ values for elastic and Prandtl-Reuss material behavior are similar up to an applied stress of about 275.8 $MN/m^2$ (40 ksi) ($p/F_{ty}=0.5$), whereas the values for an assumed Dugdale behavior are slightly higher. For stresses greater than 275.8 $MN/m^2$ (40 ksi), Dugdale and Prandtl-Reuss material behavior gives $\sqrt{J}$ values larger than the computed elastic values. At an applied stress of 413.7 $MN/m^2$ (60 ksi) ($p/F_{ty}=0.845$), the $\sqrt{J}$ values given by Dugdale and Prandtl-Reuss material behavior are about 18% higher than those assuming elastic material behavior.

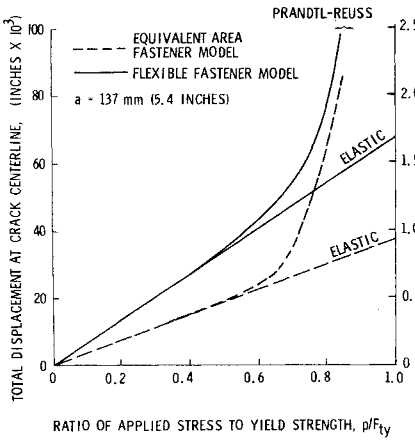


**Fig. 7 Crack opening displacement for elastic and Prandtl-Reuss material behavior.**

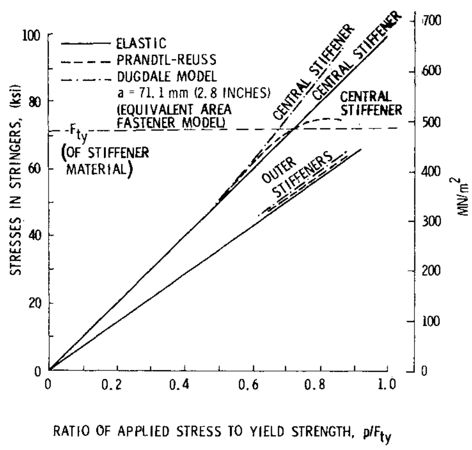


**Fig. 8 Stresses in stiffeners for assumed elastic, Prandtl-Reuss, and Dugdale model material behavior.**

The computer cost involved in an analysis based on Prandtl-Reuss material behavior assumptions is much greater than that for the Dugdale-type analysis. The $\sqrt{J}$ values given by the Dugdale analysis for stiffened panel geometries are very close to those given by Prandtl-Reuss material behavior, hence the Dugdale analysis can be used in residual strength prediction at a greatly reduced cost without sacrifice in accuracy.

### Influence of Analysis Method on Crack Openings

The crack opening at the center of the crack assuming elastic and Prandtl-Reuss material behavior for two fastener models is shown in Fig. 7. The Prandtl-Reuss material behavior crack openings are similar to those given by elastic behavior; however, at higher applied stresses, assuming Prandtl-Reuss behavior gives extremely large crack openings compared to elastic behavior. As expected (see Fig. 3), the crack openings given by the flexible fastener model are much larger than those given by the equivalent area model. At an applied stress of 60% of yield, the flexible fastener model gives crack openings 75% higher than the equivalent area model. However, the difference between crack openings given by the two models decreases with increasing load caused primarily by large plastic deformations at higher stresses.

### Influence of Analysis Method on Stiffener Stresses

The stresses in the central and outer stiffeners based on elastic, Dugdale, and Prandtl-Reuss material behavior are shown in Fig. 8 for a half-crack length of 71.1 mm (2.8 in.). The stresses in the outer stiffener appear to be independent of the type of analysis employed. The Dugdale analysis gives higher stresses in the central stiffener than those obtained

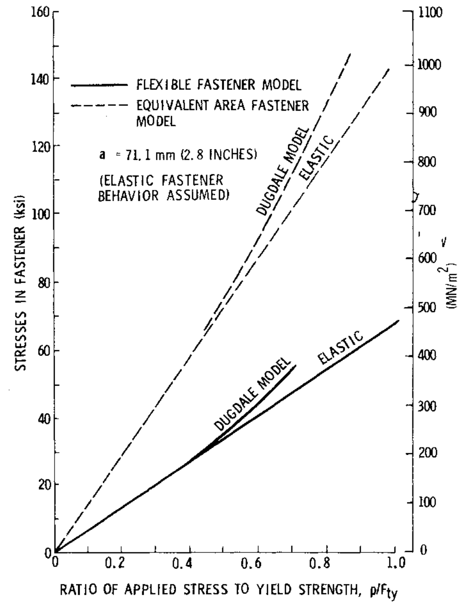


**Fig. 9 Stresses in fastener above crack plane for elastic and Dugdale model material behavior.**

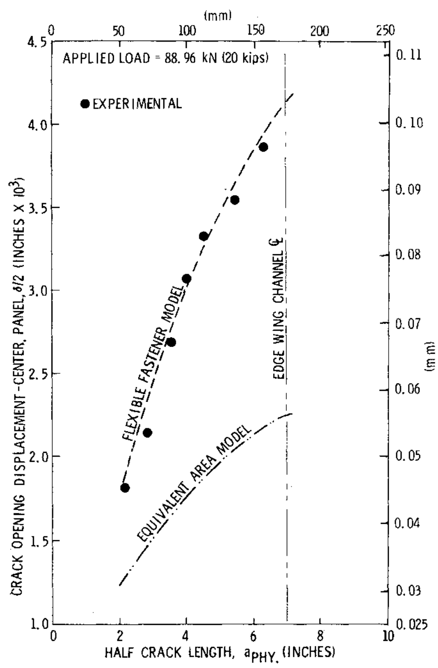


**Fig. 10 Comparison of analytical and experimental crack openings at center of the crack.**

from elastic and Prandtl-Reuss assumptions. Prandtl-Reuss and elastic behavior are similar up to 70% of $F_{ty}$, or the point where the stress in the central stiffener reaches yield stress.

### Influence of Analysis Method on Fastener Shear Stresses

The stresses in the fastener immediately above the crack plane, assuming elastic fastener behavior, are shown in Fig. 9, for elastic and Dugdale-type elastic-plastic analysis, using the two fastener models. Fastener stresses given by the Dugdale-type analysis are higher than those given by elastic analysis for both fastener models. For example, at 70% of the yield stress, the stresses given by the Dugdale-type analysis are about 12% higher than those given by elastic analysis.

## Comparison of Analytical and Experimental Results

The simulated wing channel stiffened panel shown in Fig. 1 was fabricated and tested in a 880.6-kN (200-kip) MTS

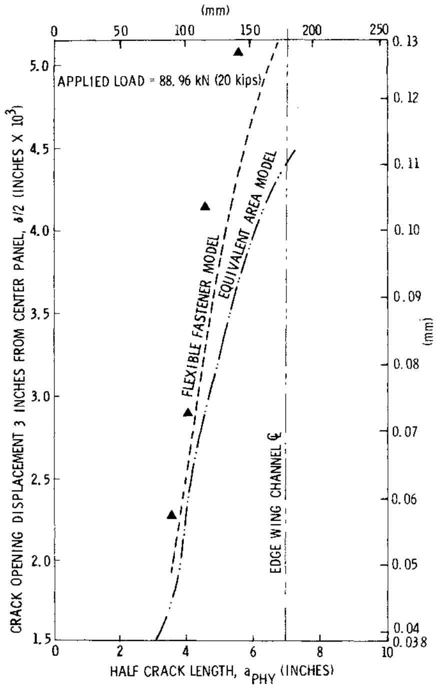


**Fig. 11 Comparison of analytical and experimental crack openings 76mm from the center of the crack.**

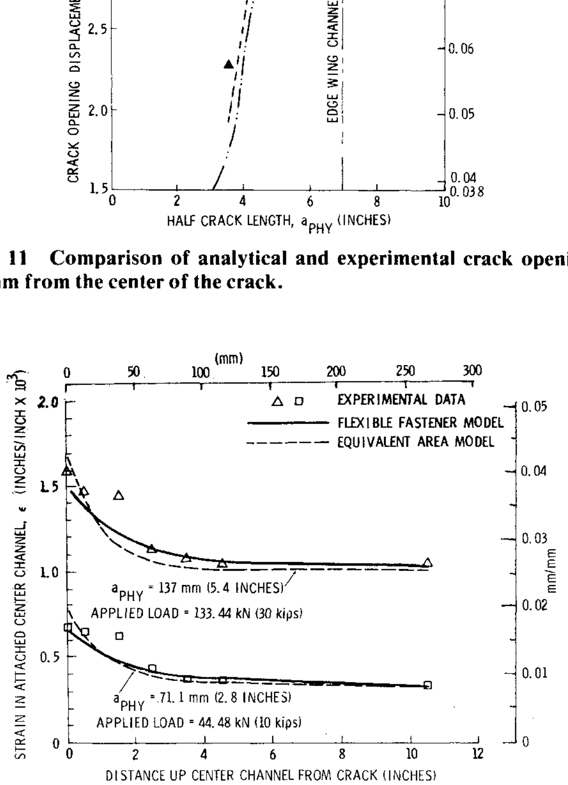


**Fig. 12 Comparison of analytical and experimental strains in central channel.**

machine. Three MTS, clip-type deflection gages were used to record crack openings at the panel centerline and 3 in. to either side of the center on the nonstiffened side. The panel was also strain-gaged to obtain strains in the stiffeners at various crack lengths at a fixed applied stress.

### Comparison of Analytical and Experimental Crack Openings

The comparison of experimentally obtained crack openings at the center of the crack and those obtained from the finite-element analysis using the two fastener models is shown in Fig. 10. It is seen that the crack openings obtained from the flexible fastener model agree remarkably well with experimental results. The crack openings at 76.2 mm (3 in.) from the centerline of the crack for various crack lengths are shown in Fig. 11. The flexible fastener model once again agrees reasonably well with experimental results (within 13% at the longest crack length).

### Comparison of Analytical and Experimental Strains in the Stiffener

A comparison of the elastic strains in the central stiffener obtained experimentally and from finite element analysis (flexible fastener and equivalent area models) is shown in Fig.

**Table 2 Comparison of residual strength of riveted, bolted, and bonded stiffened panels**

| Type of panel | Half-crack length, mm (in.) | Residual strength, $MN/m^2$ (ksi) |
|---|---|---|
| Riveted | 54.61 (2.15) | 256.84 (37.25) |
| | 112.40 (4.43) | 179.13 (25.98) |
| Bolted | 52.07 (2.05) | 250.9 (36.39) |
| | 116.84 (4.60) | 182.09 (26.41) |
| Bonded | 51.56 (2.03) | 289.79 (42.03) |
| | 107.19 (4.22) | 199.82 (28.98) |

12 for two half-crack lengths of 71.1 mm and 137 mm (2.8 in. and 5.4 in.). It is seen that the flexible fastener model provides a better fit to both data sets.

## Other Factors Influencing Residual Strength

The influence of fastener modeling and method of analysis on residual strength has already been discussed. However, there are several other factors (such as biaxial and shear loading, crack tip buckling, etc.) that also influence residual strength prediction.

In Ref. 2 it was shown that for stiffened panels, biaxial loading increases the elastic stress intensity factor, crack openings, and $\sqrt{J}$ values for elastic-plastic analysis. This effect is contrary to that observed for biaxially loaded, unstiffened panels. Similar effects have been observed by Swift.[4] This is primarily due to less load transfer occurring between the skin and the stiffener when biaxial loads are applied. The experimental results showed a decreasing residual strength with positive (tensile load in both directions) biaxial loads.

In-plane buckling of the sheet caused by the presence of the crack is known to affect the residual strength of center-cracked tension panels. In structural arrangements (stiffened skin), with the crack located between stiffeners, and sufficiently long cracks at stiffeners, a loss in residual strength can occur. The proposed analysis method does not permit crack tip buckling. Nevertheless, the fracture or failure critierion can allow for this situation by allowing a reduction in the resistance curve over that of the unbuckled state.

The residual strength of a stiffened panel is also affected by the method used to fasten the stiffeners to the skin, i.e., riveting, bolting, or bonding. For the *z*-stiffened panel geometries of Ref. 2, it was shown that the residual strengths of riveted and bolted panels are similar (indicated in Table 2). The residual strength of bonded panels was greater than that of riveted or bolted panels due to more load transfer taking place to the stiffeners, resulting in smaller crack-driving forces in the skin.

## Conclusions

Assumptions regarding fastener flexibility have been shown to have considerable influence on crack opening and load transfer in a stiffened structure. Since this will influence the residual strength prediction, fastener flexibility should be taken into consideration. The method of analysis has been shown to have considerable influence on $\sqrt{J}$ values, which will, in turn, influence the residual strength prediction. The $\sqrt{J}$ values obtained with Dugdale or Prandtl-Reuss material behavior are similar for stiffened panel geometries. Hence, a Dugdale-type analysis can be used for residual strength prediction at a much reduced cost.

## Acknowledgments

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